**Appendix A: the proof of Theorem 3.**

*Theorem 3:* Let (*N*, *M*0) be S4R*U* model of ℵ(*RU*). By solving ILP1, we obtain an MPAC π where ℜ(π) = {*r* ∈ *PR* | *vr* = 1} and ℑ(π) = {*t* ∈ *T* | *zt* = 1}, and π can lead to a BM.

*Proof:* first, we intend to find an A-circuit composed of all nodes in *T*1 = {*t* ∈ *T* | *zt* = 1} and *R*1 = {*r* ∈ *PR* | *vr* = 1}. Let *T*2 and *R*2 be the set of transitions and resource places, respectively. Initially, set *T*2 = *R*2 = ∅. Let *r*1 ∈ (*R*1 \ *R*2). By (10), there is a transition *t*1 ∈ *T*1 such that *t*1 ∈ *H*(*r*1)•. Let *p*1 = (*A*)*t*1. Then *a*1 *= p*1*t*1 is an SA-path w.r.t. resource *r*1. Since *zt*1 = 1, by (12) we know that ∃*r*2 ∈ •*t*1 ∩ *R*1 such that *vr*2 = 1. Similarly, ∃ *t*1 ∈ *T*1 such that *t*2 ∈ *H*(*r*2)• and *a*2 *= p*2*t*2 is an SA-path w.r.t. resource *r*2 where *p*2 = (*A*)*t*2. It is easy to check that *a*2 can reach *a*1, i.e., *a*1← *a*2. Set *T*2 = *T*2 ∪ {*t*1, *t*2} and *R*2 = {*r*1, *r*2}. Repeat the above procedure for each resource in *R*1 \ *R*2. When this process terminates, *R*1 = *R*2 and we can obtain an A-circuit θ = *a*1*a*2…*ak* w.r.t. resource set *R*1, i.e., ℜ(θ) = *R*1. If *T*1 \ ℑ(θ) ≠ ∅, let *t* ∈ *T*1 \ ℑ(θ). By (19) −(12), we have i) ∃*r*′ ∈ Γ((*A*)*t*) ∩ ℜ(θ), i.e., *a* = *pt* is a SA-path w.r.t. resource *r*′ where *p* = (*A*)*t*, and ii) ∃ *r*′′ ∈ •*t* ∩ ℜ(θ). Let *a*′ and *a*′′ be two SA-paths contained in θ and associated with resources *r* and *r′*, respectively. Then *a*′′ ← *a* ← *a*′ and β = *a*′′*aa*′ is an A-chain. Thus, a new A-circuit θ1 can be established by combining θ and β. Set *T*1 = *T*1 ∪ {*t*}. Repeat this process for each transition in *T*1 \ ℑ(θ). Finally, we can obtain an A-circuit π with ℜ(π) = *R*1 and ℑ(π) = *T*1.

Next, we intend to show that A-circuit π is *perfect*. Note that *PS* is the set of split places. ∀*p* ∈ *PS*, constraint (14) guarantees that *p* ∈ *P*(π) if ∃*t* ∈ *p*•∩ℑ(π); (13) implies that *p* ∈ *P*(π) only if all transitions in *p*• belong to ℑ(π). In other words, (13) and (14) enforce the perfectness of π, i.e., ((*A*)*t*)• ⊆ ℑ(π) if *t* ∈ ℑ(π). Moreover, the maximum of π is ensured by the objective function (8). Therefore, π is an MPAC.

Finally, note that there is a marking *M* ∈ *MU* + [*Nu*]*y* such that (15)−(18) are satisfied. Then by (23), we have

Π*M*(*r*)−Σ*p*∈*H*(*r*)∩*P*(π)*Xr*(*p*)*M*(*p*)≤min*t*∈*r*•∩ℑ(π)*W*(*r*, *t*) − 1, ∀*r* ∈ ℜ(π)

Then according to Definition 9 *M* is a BM w.r.t π. Therefore, the MPAC π where ℜ(π) = {*r* ∈ *PR* | *vr* = 1} and ℑ(π) = {*t* ∈ *T* | *zt* = 1} can lead to a BM. ♣

**Appendix B: the proof of Lemma 5.**

*Lemma 5:* The following inequality holds for each reachable marking *M* of controlled net (*C*[π], *M*[π]) = (*NU*, *MU*) ⊗ (*C*π, *M*π).

*M*(*p*π) + Σ*p*∈*P*(π)*M*(*p*) = *X*(π) − 1

*Proof*: By definition, the monitor (*C*π, *M*π) only control the number of tokens in *P*(π). Only transitions in *D*π ∪ *E*π ∪ {*tfi*, *tri* | *pi* ∈ *PU* ∩ *P*(π)} would affect the configuration Σ*p*∈*P*(π)*M*(*p*). Now we analyze the influence of firing these transitions.

When a transition *t* ∈ *D*π (resp. *t* ∈ *E*π) is fired, the value of configuration Σ*p*∈*P*(π)*M*(*p*) is decreased (resp. increased) by one, but the number of tokens in the control place *p*π is increased (resp. decreased) by the same amount due the arc (*p*π, *t*) (resp. (*t*, *p*π)). For each place *pi* ∈ *PU* ∩ *P*(π), although the firing of *tfi* (resp. *tri*) would lead the decrease (resp. increase) of token sum in *P*(π), the token account in *p*π is increased (resp. decreased) by the same value owing to the arcs (*tfi*, *p*π) (resp. (*p*π, *tri*)). Thus, Σ*p*∈*P*(π)*M*(*p*) + *M*(*p*π) is not changed after firing any transition in *D*π ∪ *E*π ∪ {*tfi*, *tri* | *pi* ∈ *PU* ∩ *P*(π)}.Thus, we have

*M*(*p*π) + Σ*p*∈*PAM*(*p*) = *M*[π](*p*π) + Σ*p*∈*PAM*[π](*p*) = *X*(π) − 1

Therefore, the underlying enquiry holds under each reachable marking of (*C*[π], *M*[π]). ♣